ACCESSIBILITY AND BEHAVIORAL ANALYSIS OF A DAIRY PLANT USING RPGT

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Abstract:

In this paper behavioral analysis of a single unit system under going degradation after complete failure using Regenerative Point Graphical Technique (RPGT) is discussed. Initially the unit is working at full capacity which may have two types offailures, one is direct and second one is through partial failure mode. There is a single server (repairman) who inspects and repairs the unit on each failure. Oncomplete failure the unit cannot be restored to its original capacity. On each repair unit under goes degradation if the server reports that unit is not repairable then it is replaced by a new one, which follows a general distribution. Fuzzy concept is used to determine failure/working state of a unit. Taking failure rates exponential, repair rates general and taking into consideration various probabilities, a transition diagram of the system is developed to determine Primary circuits, Secondary circuits & Tertiary circuits and Base state. Problem is formulated and solved using RPGT to determine system parameters. System behavior is discussed with the help of graphs and tables.

Keywords: Availability, Reliability, Primary Circuits, Secondary Circuits, Tertiary Circuits, Degraded state, Regenerative Point Graphical Technique (RPGT), Mean Time to System Failure, Busyperiod of the Server, Expected No. of Server's Visits, Fuzzy Logic, Steady State.

Introduction:

Ancient India has contributed a great deal to the world mathematical ocean. The country also witnessed the growth of Mathematics from 3000 B.C.E to the Modern date throwing out many important discoveries, ideas and procedures, which became the base or the working model for other countries, continents or civilizations Mathematical works. Many important discoveries from ZERO, PI to the solution of indeterminate equations were done in this era. This era also witnessed the gem of knowledge and intelligent Mathematicians like Aryabhata, Brahmagupta, Bhaskaracharya etc. whose names can't be erased out from Indian history. Arithmetic, Algebra and geometry were developed in this classical era. The decimal number which are used by current generation is the gift of this era only. India has contributed a lot in the development of modern mathematics but most of ancient work is still unknown to the world as it is in cryptic language in the form of shlokas in Sanskrit. Aryabhata was famous for giving zero but his important contribution of pi value, diameter of earth up to decimal accuracy, sine table etc.

Assumptions and Notations: – The following assumptions and notations are taken: –

1. A single repair facility is available.
2. The distributions of failure times and repair times are exponential and general respectively and also different. Failures and repairs are statistically independent.
3. Repair is imperfect and repaired system is not good as new one on complete failure.
4. Nothing can fail when the system is in failed state.
5. The system is discussed for steady-state conditions.
6. Replacement of Un-repairable unit and repair facility is immediate.

$\overline{cycle}$: A circuit formed through un-failed states.

m-cvcle : A circuit (may be formed through regenerative or non-regenerative / failed state) whose terminals are at the regenerative state m.

m-$\overline{cycle}$

$(i \xrightarrow{S_r} j)$: A circuit (may be formed through un-failed regenerative or non-regenerative state) whose terminals are at the regenerative m state.

$(\xi \xrightarrow{sff} i)$: A directed simple failure free path from ξ -state to i-state.

Vm,m : Probability factor of the state m reachable from the terminal state m of the m-cycle.

V$\overline{m,m}$ Ri: Probability factor of the state m reachable from the terminal state m of the m-$\overline{cycle}$.

Ai (t) : Reliability of the system at time t, given that the system entered the un-failed regenerative state 'i' at t = 0.

Bi (t) : Probability of the system in up time at time 't', given that the system entered regenerative state 'i' at t = 0.

Vi (t)

Wi (t)

μ_i : Reliability that the server is busy for doing a particulars job at time 't'; given that the system entered regenerative state 'i' at t = 0.

ni : Expected waiting time spent while doing a given job, given that the system entered $\eta_i = W_i^*(0)$.

Taking into consideration the above assumptions and notations the Transition Diagram of the system is given in Figure.

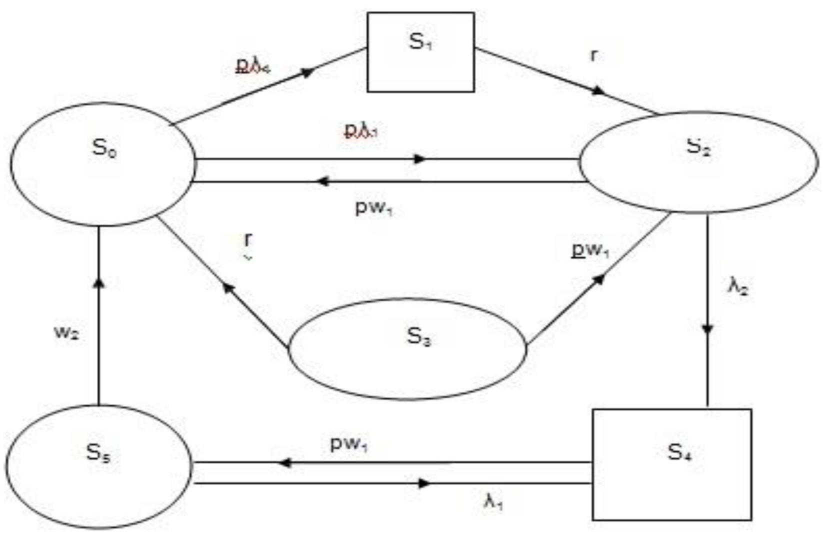


Figure 1.1

Analysis of States

State	Symbol	Model
Regenerative State/Point	⊙	0-5
Up-state	○	0,5
Failed State	□	1,4
Reduced State	⬭	2,3

Primary, Secondary & Tertiary Circuits at the various vertices.

Vertex i	Primary Circuits (CL1)	Secondary Circuits (CL2)
0	(0,1,2)	Nil
1	(1,2,0)	Nil
2	(2,0,1)	Nil

Table 1.1

From the table 1.1 we see that at working state '0' there are maximum number of primary circuits, hence state '0' is the base state.

Primary, Secondary, Tertiary Circuits w.r.t. the Simple Paths (Base-State '0')

Vertex j	$\left(0 \overset{S_r}{\rightarrow} j\right)$: (P0)	(P1)
0	$\left(\mathbf{0} \overset{S_1}{\rightarrow} \mathbf{0}\right)$:{0,1,0}	Nil
1	$\left(\mathbf{0} \overset{S_1}{\rightarrow} \mathbf{1}\right)$:{0,1}	Nil
2	$\left(\mathbf{3} \overset{S_1}{\rightarrow} \mathbf{2}\right)$:{0,1,2}	Nil

Table 1.2

Transition Probabilities

Transition Probabilities

$q_{i,j}^{(t)}$	$P_{ij} = q^*_{i,j}{}^{(t)}$
$\boldsymbol{q_{0,1}} = \boldsymbol{\lambda_1 e^{-\lambda_1 t}}$	$\boldsymbol{p_{0,1}} = 1$
$\boldsymbol{q_{1,2}} = \boldsymbol{\lambda_2 e^{-\lambda_2 t}}$	$\boldsymbol{p_{1,2}} = 1$

$q_{2,0} = w_1 e^{-w_1 t}$ $\quad$ $p_{2,0} = 1$

Table 1.3

Mean Sojourn Times

Mean Sojourn Times

$R_i(t)$	$\mu_i = R_i^*(0)$
$R_0^{(t)} = e^{-\lambda_1 t}$	$\mu_0 = 1/\lambda_1$
$R_1^{(t)} = e^{-\lambda_2 t}$	$\mu_1 = 1/\lambda_2$
$R_2^{(t)} = e^{-w_1 t}$	$\mu_2 = 1/w_1$

Table 1.4

Evaluation of Parameters: - The Mean time to system failure and all the key parameters of the system under steady state conditions are evaluated, applying Regenerative Point Graphical Technique (RPGT) and using '0' as the base-state of the system as under:

The transition probability factors of all the reachable states from the base state 'ξ' = '0' are:

Probabilities from state '0' to different vertices are given as

$V_{0,0} = (0,1,2,0) = p_{0,1}\, p_{1,2} p_{2,0} = 1$

$V_{0,1} = (0,1) = p_{0,1} = 1$

$V_{0,2} = (0,1,2)$

$= p_{0,1}\, p_{1,2} = 1$

MTSF(T_0): The regenerative un-failed states to which the system can transit (initial state '0'), before entering any failed state are: 'i' = 0, 1, taking 'ξ' = '0'.

$$\text{MTSF } (T_0) = \left[\sum_{i,sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr(sff)} i\right)\right\}\mu i}{\Pi_{m_1\neq\xi}\left\{1\text{-}V_{\overline{m_1 m_1}}\right\}}\right\}\right] \div \left[1\text{-}\sum_{sr}\left\{\frac{\left\{pr\left(\xi \xrightarrow{sr(sff)} \xi\right)\right\}}{\Pi_{m_2\neq\xi}\left\{1\text{-}V_{\overline{m_2 m_2}}\right\}}\right\}\right]$$

$$= (V_{0,0}\mu_0 + V_{0,1}\mu_1)/(V_{0,0}\mu_0 + V_{0,1}\mu_1 + V_{0,2}\mu_2)$$

Availability of the System: The regenerative states at which the system is available are 'j' = 0, 1 and the regenerative states are 'i' = 0, 1, 2 taking 'ξ' = '0' the total fraction of time for which the system is available is given by

$$A_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\to}j)\}fj,\mu j}{\Pi_{m_1\neq\xi}\{1\text{-}V_{\overline{m_1m_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\Pi_{m_2\neq\xi}\{1\text{-}V_{\overline{m_2m_2}}\}}\right\}\right]$$

$$A_0 = \left[\sum_j V_{\xi,j}, f_j, \mu_j\right] \div \left[\sum_i V_{\xi,i}, f_j, \mu_i^1\right] = (V_{0,0}f_0\mu_0+V_{0,1}f_1\mu_1)/(V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2)$$

Proportional Busy Busy Period of the Server: The regenerative states where server 'j' = 2 and regenerative states are 'i' = 0 to 2taking ξ = '0', the total fraction of time for which the server remains busy is

$$B_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\to}j)\},nj}{\Pi_{m_1\neq\xi}\{1\text{-}V_{\overline{m_1m_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\Pi_{m_2\neq\xi}\{1\text{-}V_{\overline{m_2m_2}}\}}\right\}\right]$$

$$= \left[\sum_j V_{\xi,j}, n_j\right] \div \left[\sum_i V_{\xi,i}, \mu_i^1\right]$$

$$= (V_{2,0}\mu_2)/(V_{0,0}\,\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2)$$

Expected Number of Inspections by the repair man: The regenerative states where the repairman does this job j = 2 the regenerative states are i = 0 to 2, Taking 'ξ' = '0', the number of visit by the repair man is given by

$$V_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\to}j)\}}{\Pi_{k_1\neq\xi}\{1\text{-}V_{\overline{k_1k_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\Pi_{k_2\neq\xi}\{1\text{-}V_{\overline{k_2k_2}}\}}\right\}\right]$$

$$V_0 = \left[\sum_j V_{\xi,j}\right] \div \left[\sum_i V_{\xi,i}, \mu_i^1\right]$$

$$V_0 = (V_{2,0}\mu_2)/(V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2)$$

Illustration: When failure and repair rates are equal

MTSF (T_0) $=[(1/\lambda_1)+(1/\lambda_2)]/[(1/\lambda_1)+(1/\lambda_2)+(1/w_1)]$

$= [2w/(2w+\lambda)]$

MTSF Table

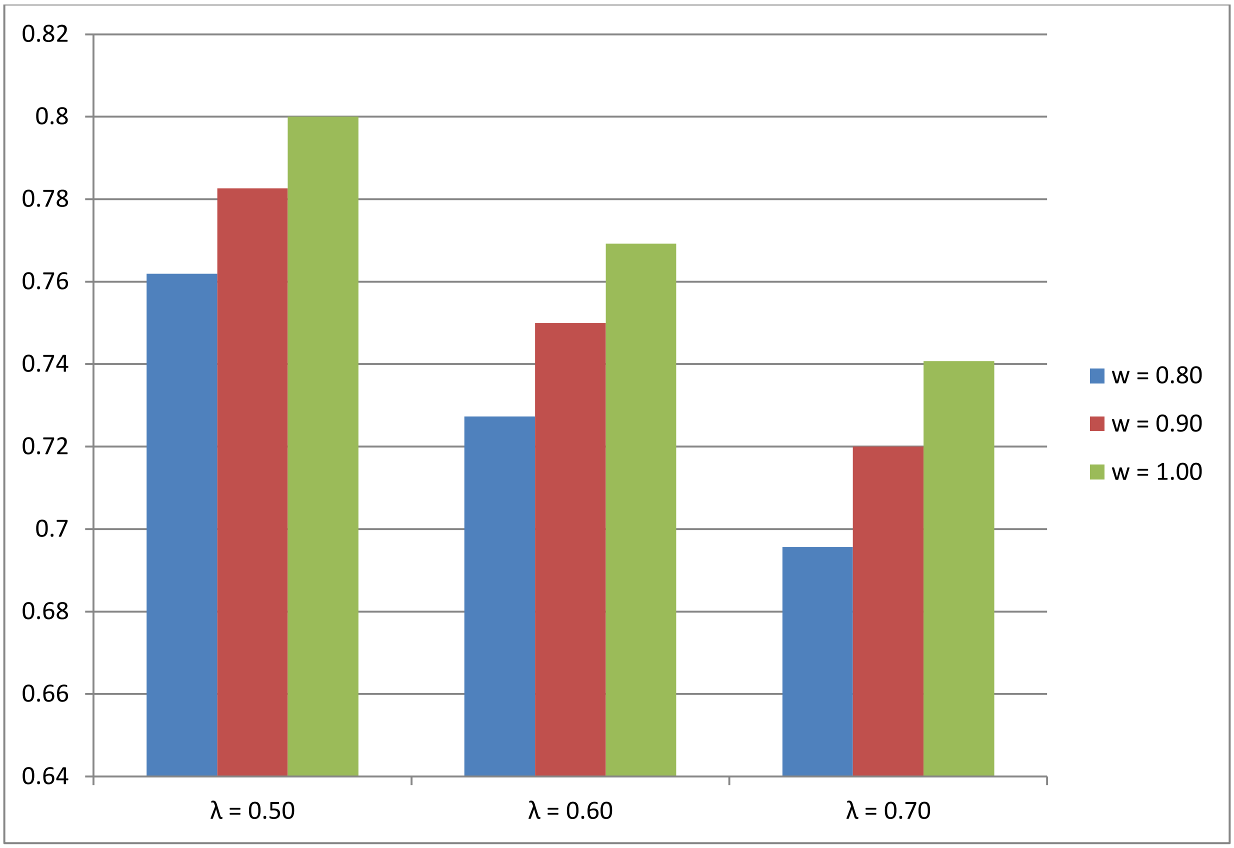
T_0	w = 0.80	w = 0.90	w = 1.00
λ = 0.50	0.761904	0.782609	0.800000

λ = 0.60	0.727272	0.750000	0.769230
λ = 0.70	0.695652	0.720000	0.740740

Table 1.5

MTSF Graph

Figure 1.2



Availability of the System (A_0)

Availability of the System Table

A_0	w = 0.80	w = 0.90	w = 1.00
λ = 0.50	0.761904	0.782609	0.800000
λ = 0.60	0.727272	0.750000	0.769230

$\lambda = 0.70$	0.695652	0.720000	0.740740

Table 1.6

Availability of the System Graph

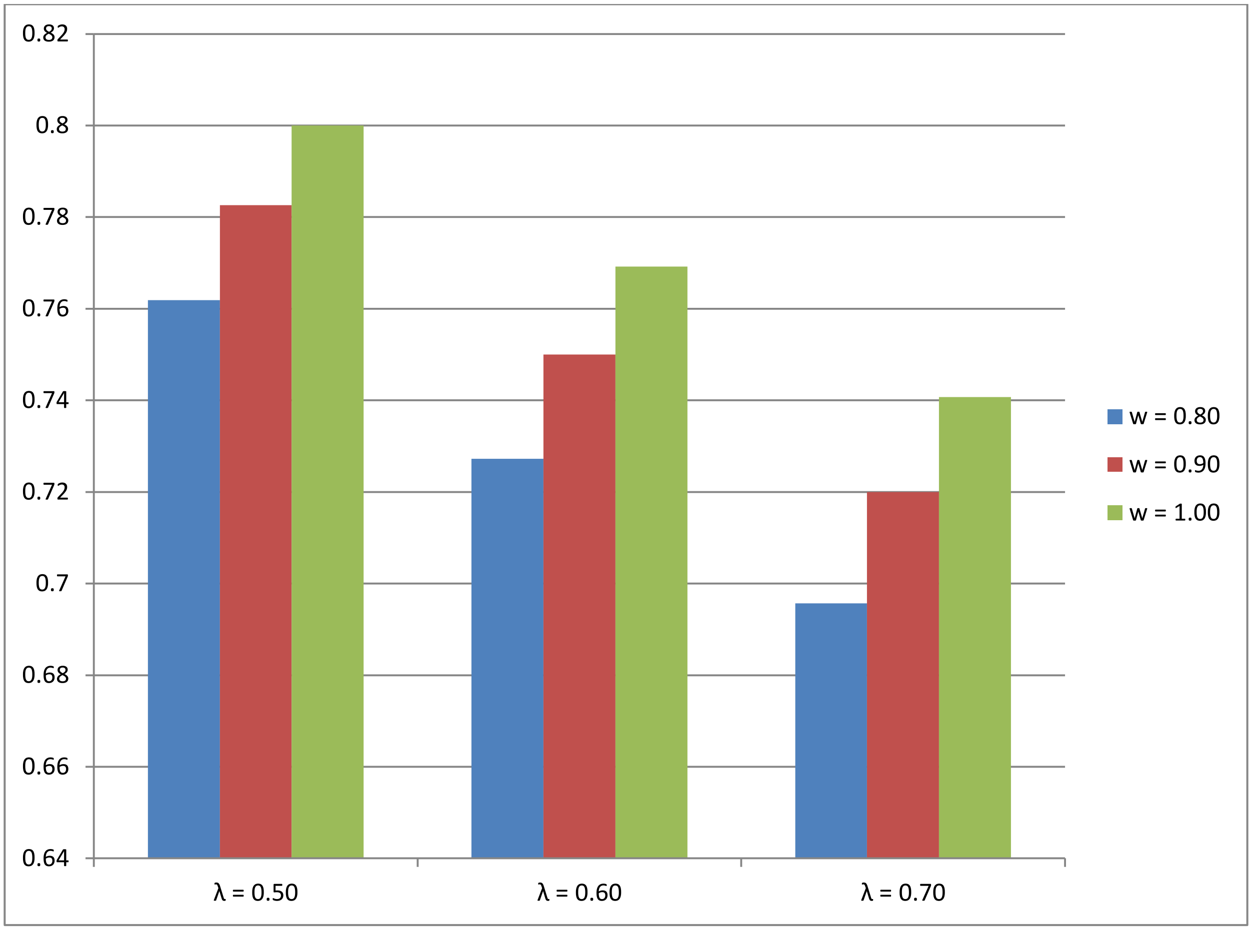


Figure 1.3

Fractional Busy Period of the Server(B_0) in Unit Time:

$$(\mathbf{B_0}) = [(1/w_1)/\{(1/\lambda_1)+(1/\lambda_2)+(1/w_1)\}]$$

$$= [\lambda/(2w+\lambda)]$$

Busy Period of the Server Table

B_0	w = 0.80	w = 0.90	w = 1.00
$\lambda = 0.50$	0.238095	0.217391	0.200000
$\lambda = 0.60$	0.272727	0.250000	0.230769

λ = 0.70	0.304348	0.280000	0.259259

Table 1.7

Busy Period of the Server Graph

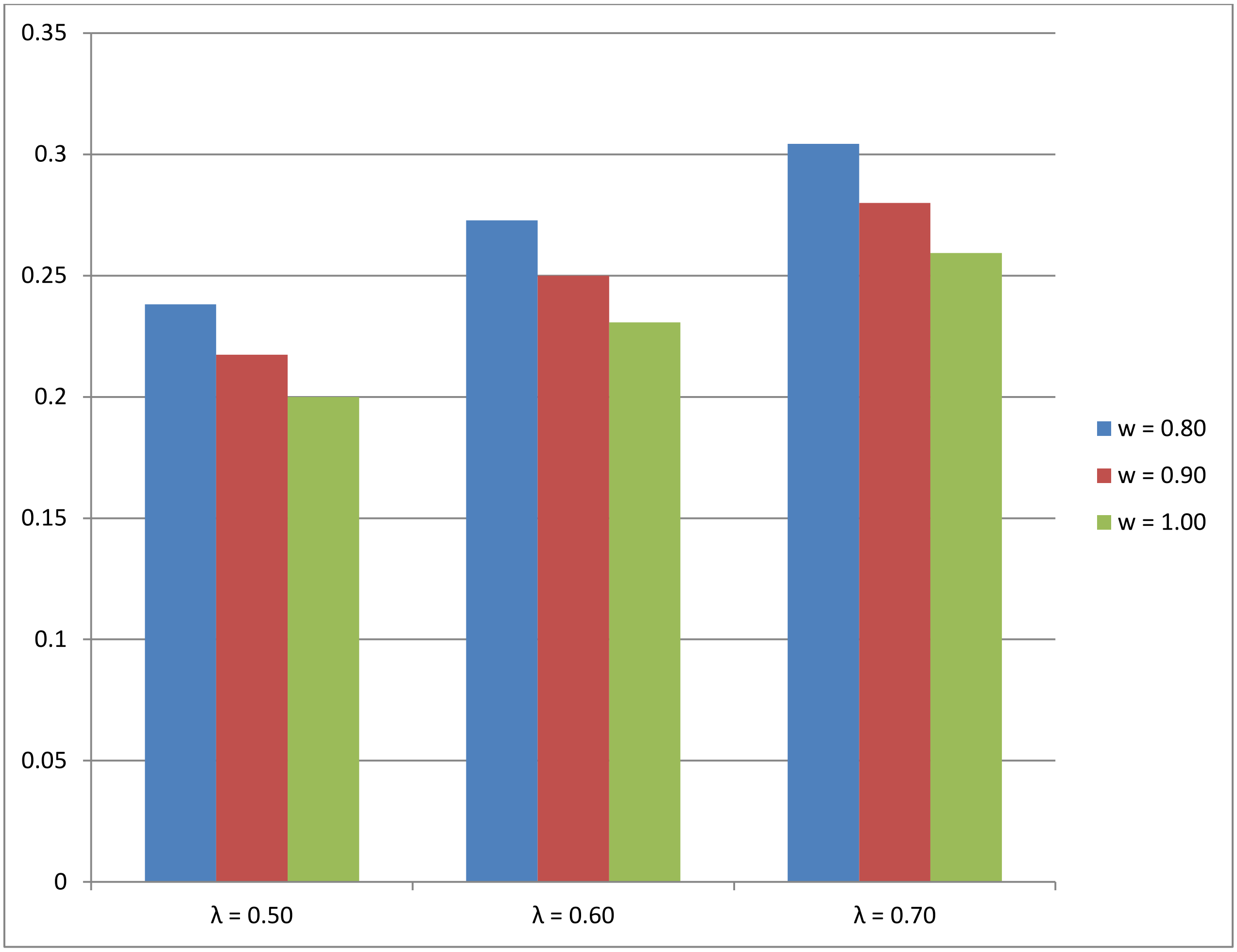


Figure 1.4

Expected Fractional Number of Server's Visits(V_0) in Unit Time:

Expected Number of Server's Visits Table

V_0	w = 0.80	w = 0.90	w = 1.00
λ = 0.50	0.238095	0.217391	0.200000
λ = 0.60	0.272727	0.250000	0.230769
λ = 0.70	0.304348	0.280000	0.259259

Table 1.8

Expected Number of Server's Visits Graph

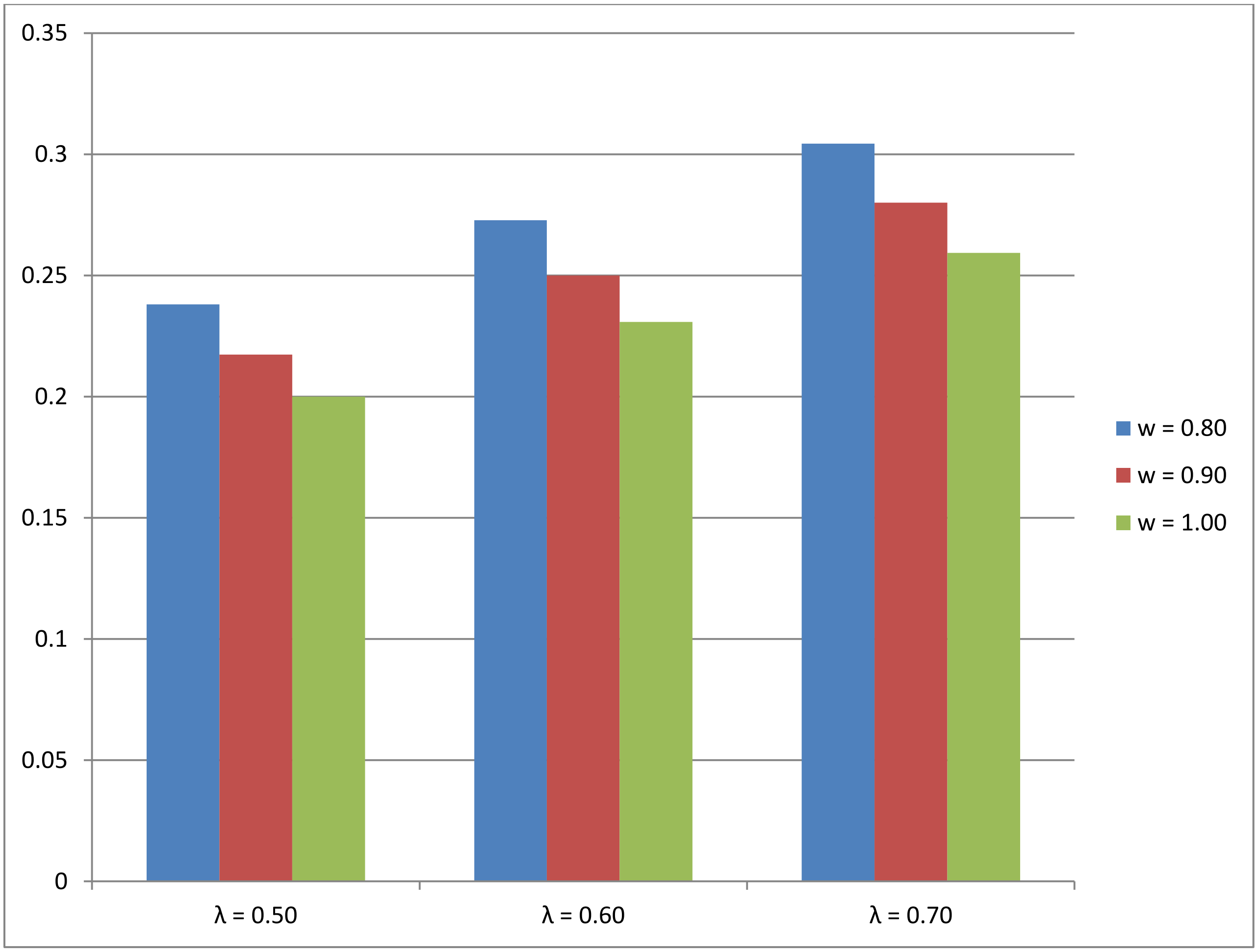


Figure 1.5

Profit Function

$= A_0R_0-(B_0R_1+V_0R_2)$

$= A_0R_0-B_0R_1-V_0R_2$

Where

A_0 = Availability of System

B_0 = Busy Period of Server

V_0 = Expected Number of Inspection by the Repair Man

R_0 = Revenue

R_1 = Busy Period per Unit

R_2 = Per Visit Cost

R_0 = 1000

$R_1 = 50$

$R_2 = 100$

Profit= $[2000w/(2w+\lambda)]-[50\lambda/(2w+\lambda)]-[100\lambda/(2w+\lambda)]$

$= [2000w-150\lambda]/[(2w+\lambda)]$

	w = 0.5	w = 0.6	w = 0.7
λ = 0.5	726.1905	750.0000	770.0000
λ = 0.6	686.3636	712.5000	734.6154
λ = 0.7	598.0000	678.0000	701.8519

Profit Table 1.9

Profit Function Graph

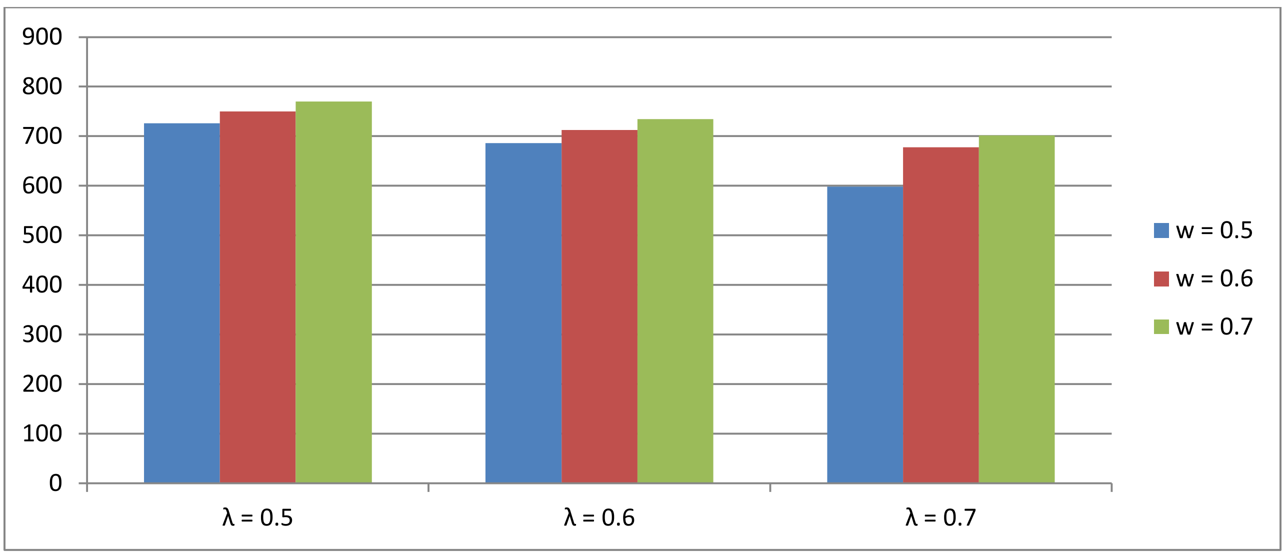


Figure 1.6

Conclusion:

Evolution of man started instinctively wherein he started seeing a pattern and geometry in the formation and cyclical form of Mother Nature which he considered and held on to as the guiding stick and started his journey of exploration. The world knows Indian civilization was the oldest, we Indians should be proud of the progress we had made in various fields like Astronomy, Astrology, Technology, Medicines, architecture, construction and many more.

Ganita was one of the chapters or portfolios of ancient treatise. The advancement today in the field of Mathematics is largely due to prominent contribution from India. In today's scenario, the world tips in favor of more prosperous countries. The knowledge pools from over the world, work for more advanced countries, the western world today holds the reins. The recognition and the due credit the Indian civilization and Mathematicians deserved is not given though the point of concern is lot of Indians are so blinded by the western world that we ourselves either do not give or do not want to give the true credit and credibility to our heritage. Whatever credit is being given is due to the recognition now, as the whole world knows still there are solutions to lot of intriguing mysteries in Indian Vedas and siddhantas.

Deep dive into it and one can find the way. An attempt is made here in the field of math but the path is complex and interwined, the entangled nature of various works done in isolation have made the deciphering and getting it out in a systematic manner has become a herculian task. This work also emphasizes that the effort here is just a miniscule one and would require many more efforts like this and a more prominent government intervention.

There are total 123 sutras in ryabhat ya only 17 of them are deciphered and proved here, and in Brahmasphuta siddhanta there are 1008 sutras out of which 27 are decrypted and proof or examples are provided in this thesis so in all 40 sutras are worked on with proof or algorithm or examples, the result which triggered was that all what we learn in elementary math is infact only a refined form of ancient Indian Math.

The sutras or verses are so powerful that it encompasses an explanation of entire mathematical formulation that we decipher in several pages.

All these work by the Indian Mathematicians leaves one pondering whether the modernization and so-called systemization has proved beneficial or has killed the advanced knowledge.

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