

A CHARACTERIZATION OF NEUTROSOPHIC INFRA B-OPEN SET IN NEUTROSOPHIC INFRA TOPOLOGICAL SPACE

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ABSTRACT

The aim of this paper is to define and study a new class of sets called neutrosophic infra b-open sets in neutrosophic infra topological space. Further we have analyse the properties of neutrosophic infra b-open sets.

Keywords:Neutrosophic infra set, Neutrosophic infra open sets, Neutrosophic infra topological spaces.

I.INTRODUCTION

In 1965, Zadeh [7] introduced fuzzy set, where each element had a degree of membership. The intuitionistic fuzzy set was introduced by K.Atanassov [4] in 1986 as a generalization of fuzzy set. The neutrosophic set concept was introduced by Smarandachein 1999 [5] by the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^{-}0,1^{+}[$ is a non-standard unit interval. The concepts of Neutrosophic Set and Neutrosophic topological Spaces was introduced by Salama&Alblowi[1] in 2012. Adel.M.AL.Odhari [2,3] introduced the concept of infra topological space and studied infra continuous and infra* continuous functions. Vaiyomathi.k and Nirmala Irudayam.F[6] introduced the notion of infra open sets and studied various properties in infra topological spaces. In this paper we introduce the definitions of neutrosophic infra open sets. And we obtain the several properties which enables us to bring out the relationship between these sets.

II.PRELIMINARIES

Definition 2.1: Let T, I, F be real standard or nonstandard subsets of $]^{-}0,1^{+}[$, with

Sup-T=t-sup, inf-T=t-inf; Sup-I=i-sup, inf-I=i-inf ; Sup-F=f-sup, inf-F=f-inf

n-sup= t-sup+ i-sup+ f-sup; n-inf= t- inf + i- inf + f- inf ,Where T, I, F are called the neutrosophic components.

Definition 2.2 [5]: Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ Where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ which represent the degree of membership function, the degree of indeterminacy, the degree of non-membership respectively of each element $x \in X$ to the set A . A neutrosophic $A = \{ \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle : x \in X \}$ can be identified to an ordered triple $\langle \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle$ in $]^{-0, 1^+}[$ on X .

Definition 2.3 [5]: Let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in X . Then

$$(i) \cap A = \{ \langle x, \wedge \mu_{A_i}(x), \wedge \sigma_{A_i}(x), \vee \gamma_{A_i}(x) \rangle : x \in X \}$$

$$(ii) \cup A = \{ \langle x, \vee \mu_{A_i}(x), \vee \sigma_{A_i}(x), \wedge \gamma_{A_i}(x) \rangle : x \in X \}$$

Definition 2.4 [1,5]: A neutrosophic topology (NT) on a nonempty set X is a family τ of neutrosophic sets in X satisfying the following axioms:

$$(i) 0_N, 1_N \in \tau$$

$$(ii) G_1 \cap G_2 \in \tau \text{ for any } G_1, G_2 \in \tau$$

$$(iii) \cup G_i \in \tau \text{ for arbitrary family } \{G_i : i \in \Lambda\} \subseteq \tau$$

In this case the ordered pair (X, τ) or simply X is called a neutrosophic topological spaces (NTS) and each neutrosophic set in τ is called a neutrosophic open set (NOS). The complement (A^c) of a NOS A in X is called a neutrosophic closed set (NCS) in X .

Definition 2.5: Let A be a neutrosophic set in a neutrosophic topological space X . Then

$$(i) Nint(A) = \cup \{G : G \text{ is a neutrosophic open set in } X \text{ and } G \subseteq A\}$$

$$(ii) Ncl(A) = \cap \{G : G \text{ is a neutrosophic closed set in } X \text{ and } G \supseteq A\}$$

Definition 2.6 [2]: Let X be any arbitrary set. An Infra -topological space on X is a collection τ_{iX} subsets of X such that the following axioms are satisfying:

$$Ax-1: \phi, X \in \tau_{iX}.$$

Ax-2: The intersection of the elements of any sub collection of τ_{iX} in X .

$$i.e) \text{ If } O_i \in \tau_{iX}, 1 \leq i \leq n \rightarrow \cap O_i \in \tau_{iX}.$$

Terminology, the order pair (X, τ_{iX}) is called infra-topological space. We simply say X is an infra space.

Definition 2.7 [2]: Let (X, τ_{iX}) be an infra-topological space and $A \subset X$. A is called an infra open set (IOS) if $A \in \tau_{iX}$ and A is called an infra-closed set (ICS) iff $X \setminus A \in \tau_{iX}$.

Definition 2.8: Let (X, τ_{iX}) be an infra topological space and $A \subset X$. The **Infra Closure Point (ICP)** of A is a set denoted by $\text{icp}(A)$ and given by : $\text{icp}(A) = \cap \{C_i : A \subseteq C_i, X - C_i \in \tau_{iX}\}$

(i.e) $\text{icp}(A)$ is the intersection of all infra closed set containing the set A .

Definition 2.9: Let (X, τ_{iX}) be an infra topological space and $A \subset X$. The **Infra Interior Point (IIP)** of A is a set denoted by $\text{iip}(A)$ and given by: $\text{iip}(A) = \cup \{O_i : O_i \subseteq A, O_i \in \tau_{iX}\}$

(i.e) $\text{iip}(A)$ is the union of all infra open set contained in the set A .

III. NEUTROSOPHIC INFRA TOPOLOGICAL SPACES

In this section, we define a neutrosophic infra set and introduce a new class of open sets called neutrosophic infra semi-open set, neutrosophic infra pre-open set, neutrosophic infra α -open set, neutrosophic infra b-open set, neutrosophic infra β -open set and study some of their properties.

Definition 3.1: Let X be a non-empty fixed set. A neutrosophic infra set (NIS) A_{NI} is an object having the form $A_{NI} = \left\{ \langle x, \mu_{A_{NI}}(x), \sigma_{A_{NI}}(x), \gamma_{A_{NI}}(x) \rangle : x \in X \right\}$ Where $\mu_{A_{NI}}(x), \sigma_{A_{NI}}(x)$ and $\gamma_{A_{NI}}(x)$ which represent the degree of membership function, the degree of indeterminacy, the degree of non-membership respectively of each element $x \in X$ to the set A_{NI} .

A neutrosophic infra set $A_{NI} = \left\{ \langle x, \mu_{A_{NI}}(x), \sigma_{A_{NI}}(x), \gamma_{A_{NI}}(x) \rangle : x \in X \right\}$ can be identified to an ordered triple $\langle \mu_{A_{NI}}(x), \sigma_{A_{NI}}(x), \gamma_{A_{NI}}(x) \rangle$ in $]^{-}0, 1^{+}[$ on X .

Definition 3.2: Let X be a non-empty fixed set. The neutrosophic infra sets A_{NI} and B_{NI} are in the form as follows:

$A_{NI} = \left\{ \langle x, \mu_{A_{NI}}(x), \sigma_{A_{NI}}(x), \gamma_{A_{NI}}(x) \rangle : x \in X \right\}$, $B_{NI} = \left\{ \langle x, \mu_{B_{NI}}(x), \sigma_{B_{NI}}(x), \gamma_{B_{NI}}(x) \rangle : x \in X \right\}$ Then

(i) The complement of the neutrosophic infra set A_{NI} (ie: (A_{NI}^c)) defined as

$$A_{NI}^c = \left\{ \langle x, \gamma_{A_{NI}}(x), \sigma_{A_{NI}}(x), \mu_{A_{NI}}(x) \rangle : x \in X \right\}$$

(ii) The subset of the neutrosophic infra sets A_{NI} and B_{NI} are defined as

$$A_{NI} \subseteq B_{NI} \Leftrightarrow \mu_{A_{NI}}(x) \leq \mu_{B_{NI}}(x), \sigma_{A_{NI}}(x) \leq \sigma_{B_{NI}}(x), \gamma_{A_{NI}}(x) \geq \gamma_{B_{NI}}(x) \text{ for all } x \in X$$

(iii) The neutrosophic infra sets $A_{NI} = B_{NI} \Leftrightarrow A_{NI} \subseteq B_{NI}$ and $B_{NI} \subseteq A_{NI}$

(iv) The intersection of the neutrosophic infra sets A_{NI} and B_{NI} are defined as

$$A_{NI} \cap B_{NI} = \left\{ \langle x, \mu_{A_{NI}}(x) \wedge \mu_{B_{NI}}(x), \sigma_{A_{NI}}(x) \wedge \sigma_{B_{NI}}(x), \gamma_{A_{NI}}(x) \vee \gamma_{B_{NI}}(x) \rangle : x \in X \right\}$$

(v) The union of the neutrosophic infra sets A_{NI} and B_{NI} are defined as

$$A_{NI} \cup B_{NI} = \left\{ \langle x, \mu_{A_{NI}}(x) \vee \mu_{B_{NI}}(x), \sigma_{A_{NI}}(x) \vee \sigma_{B_{NI}}(x), \gamma_{A_{NI}}(x) \wedge \gamma_{B_{NI}}(x) \rangle : x \in X \right\}$$

(vi) $[] A_{NI} = \left\{ \langle x, \mu_{A_{NI}}(x), \sigma_{A_{NI}}(x), 1 - \mu_{A_{NI}}(x) \rangle : x \in X \right\}$

(vii) $\langle \rangle A_{NI} = \left\{ \langle x, 1 - \gamma_{A_{NI}}(x), \sigma_{A_{NI}}(x), \gamma_{A_{NI}}(x) \rangle : x \in X \right\}$

Since our aim is to construct the tool for developing neutrosophic infra topological spaces, we must introduce the neutrosophic infra sets 0_{NI} and 1_{NI} in X as follows:

Definition 3.3: Let $0_{NI} = \{ \langle x, 0, 0, 1 \rangle : x \in X \}$ and $1_{NI} = \{ \langle x, 1, 1, 0 \rangle : x \in X \}$

Definition 3.4: A neutrosophicinfra topology (NIT) on a nonempty set X is a family τ_{NI} of neutrosophic sets in X satisfying the following axioms:

(i) $0_N, 1_N \in \tau_{NI}$

(ii) $G_1 \cap G_2 \in \tau_{NI}$ for any $G_1, G_2 \in \tau_{NI}$

In this case the ordered pair (X, τ_{NI}) or simply X is called a neutrosophic infra topological spaces (NITS) and each neutrosophic set in τ_{NI} is called a neutrosophic open set (NIOS).

The complement (A_{NI}^c) of a NIOS A_{NI} in X is called a neutrosophic closed set (NICS) in X .

Definition 3.5: Let A_{NI} be a neutrosophic infra set in a neutrosophicinfra topological space X . Then $NIint(A_{NI}) = \cup \{ F : F \text{ is a neutrosophic open set in } X \text{ and } F \subseteq A_{NI} \}$ is called the neutrosophic interior of A_{NI} ;

$NIcl(A_{NI}) = \cap \{ K : K \text{ is a neutrosophic closed set in } X \text{ and } K \supseteq A_{NI} \}$ is called the neutrosophic closure of A_{NI} .

Also here $NIcl(A_{NI})$ is NICS and $NIint(A_{NI})$ is a NIOS in X.

- (i) A_{NI} is in X if and only if $NIcl(A_{NI})$.
- (ii) A_{NI} is NICS in X if and only if $NIint(A_{NI}) = A_{NI}$

Proposition 3.6: Let (X, τ_{NI}) be an neutrosophic infra topological space and the twoneutrosophic infra sets A_{NI} and B_{NI} in X. Then the following properties hold:

- (i) $NIint(A_{NI}) \subseteq A_{NI}$, (ii) $A_{NI} \subseteq NIcl(A_{NI})$
- (iii) $A_{NI} \subseteq B_{NI} \Rightarrow NIint(A_{NI}) \subseteq NIint(B_{NI})$, (iv) $A_{NI} \subseteq B_{NI} \Rightarrow NIcl(A_{NI}) \subseteq NIcl(B_{NI})$
- (v) $NIint(NIint(A_{NI})) = NIint(A_{NI})$, (vi) $NIcl(NIcl(A_{NI})) = NIcl(A_{NI})$
- (vii) $NIint(1_{NI}) = 1_{NI}$, (viii) $NIcl(0_{NI}) = 0_{NI}$
- (ix) $NIint(A_{NI} \cup B_{NI}) = NIint(A_{NI}) \cup NIint(B_{NI})$; (x) $NIcl(A_{NI} \cap B_{NI}) = NIcl(A_{NI}) \cap NIcl(B_{NI})$

Definition 3.7: A neutrosophic infra set $A_{NI} = \{ \langle x, \mu_{A_{NI}}(x), \sigma_{A_{NI}}(x), \gamma_{A_{NI}}(x) \rangle : x \in X \}$ in neutrosophic infra topological space (X, τ_{NI}) is said to be

- (i) Neutrosophic infra semi-open set (NISOS in short) if $A_{NI} \subseteq NIcl(NIint(A_{NI}))$.
- (ii) Neutrosophic infra pre-open set (NIPOS in short) if $A_{NI} \subseteq NIint(NIcl(A_{NI}))$.
- (iii) Neutrosophicinfra α -open set (NI α OS in short) if $A_{NI} \subseteq NIint(NIcl(NIint(A_{NI}))$.
- (iv) Neutrosophic infra b-open set (NIBOS in short) if $A_{NI} \subseteq NIint(NIcl(A_{NI})) \cup NIcl(NIint(A_{NI}))$.
- (v) Neutrosophicinfra β -open set (NI β OS in short) if $A_{NI} \subseteq NIcl(NIint(NIcl(A_{NI}))$.
- (vi) Neutrosophic infra regular open set (NIROS in short) if $A_{NI} = NIint(NIcl(A_{NI}))$.

The complement of the above mentioned neutrosophic infra open sets are called their respective neutrosophic infra closed sets.

Proposition 3.8: Let A_{NI} be a neutrosophic infra set of a space (X, τ_{NI}) . Then

$$(1) NIsc(A_{NI}) = A_{NI} \cup (NIint(NIcl(A_{NI})))$$

$$NI_{sint}(A_{NI}) = A_{NI} \cap (NI_{cl}(NI_{int}(A_{NI})))$$

$$(2) NI_{pcl}(A_{NI}) = A_{NI} \cup (NI_{cl}(NI_{int}(A_{NI})))$$

$$NI_{pint}(A_{NI}) = A_{NI} \cap (NI_{int}(NI_{cl}(A_{NI})))$$

$$(3) NI_{spcl}(A_{NI}) = A_{NI} \cup (NI_{int}(NI_{cl}(NI_{int}(A_{NI}))))$$

$$NI_{spint}(A_{NI}) = A_{NI} \cap (NI_{cl}(NI_{int}(NI_{cl}(A_{NI}))))$$

$$(4) NI_{scl}(NI_{sint}(A_{NI})) = NI_{sint}(A_{NI}) \cup (NI_{int}(NI_{cl}(NI_{int}(A_{NI}))))$$

$$(5) NI_{pcl}(NI_{pint}(A_{NI})) = NI_{pint}(A_{NI}) \cup (NI_{cl}(NI_{int}(A_{NI})))$$

$$(6) NI_{spcl}(NI_{spint}(A_{NI})) = NI_{spint}(NI_{spcl}(A_{NI}))$$

Theorem 3.9: In a neutrosophic infra topological space (X, τ_{NI})

(i) Every neutrosophic *infra semi-open* set is neutrosophic *infra b-open* set.

(ii) Every neutrosophic *infra pre-open* set is neutrosophic *infra b-open* set.

Proof: The result is Obvious.

Example 3.10: Let (X, τ_{NIX}) be a neutrosophic infra topological space and $X = \{a, b\}$. Define the neutrosophic infra sets A, B and C as follows:

$$A = \left\langle x, \frac{a}{(0.3, 0.2, 0.5)}, \frac{b}{(0.4, 0.4, 0.7)} \right\rangle, \quad B = \left\langle x, \frac{a}{(0.4, 0.5, 0.7)}, \frac{b}{(0.5, 0.6, 0.8)} \right\rangle \text{ and}$$

$$C = \left\langle x, \frac{a}{(0.5, 0.5, 0.6)}, \frac{b}{(0.6, 0.6, 0.7)} \right\rangle. \text{ Then the families } \tau_{NIX} = \{0_{NI}, 1_{NI}, A, B, A \cap B\} \text{ is}$$

a neutrosophic infra topology on X. Here C is neutrosophic *infra b-open* set but not neutrosophic *infra pre-open* set.

Example 3.10: Let (X, τ_{NIX}) be a neutrosophic infra topological space and $X = \{a, b\}$. Define the neutrosophic infra sets A, B and C as follows:

$$A = \left\langle x, \frac{a}{(0.4, 0.5, 0.5)}, \frac{b}{(0.5, 0.6, 0.5)} \right\rangle, \quad B = \left\langle x, \frac{a}{(0.3, 0.5, 0.6)}, \frac{b}{(0.4, 0.5, 0.7)} \right\rangle \text{ and}$$

$$C = \left\langle x, \frac{a}{(0.4, 0.5, 0.4)}, \frac{b}{(0.5, 0.5, 0.5)} \right\rangle. \text{ Then the families } \tau_{NIX} = \{0_{NI}, 1_{NI}, A, B, A \cap B\} \text{ is}$$

anerosophic infra topology on X. Here C is neutrosophic infra b-open set but not neutrosophic infra semi-open set.

Proposition 3.11: Let A_{NI} be anerosophic infra set of a space (X, τ_{NI}) . Then the following are equivalent:

- a) A_{NI} is neutrosophic infra b-openset.
- b) $A_{NI} = NI\text{pint}(A_{NI}) \cup NIsint(A_{NI})$
- c) $A_{NI} \subseteq (NI\text{pcl}(NI\text{pint}(A_{NI})))$

Proof: Let $A_{NI} \subseteq (X, \tau_{NI})$.

(a) \Rightarrow (b) Let A_{NI} be a neutrosophic infra b-openset. (i.e) $A_{NI} \subseteq NI\text{int}(NI\text{cl}(A_{NI})) \cup NI\text{cl}(NI\text{int}(A_{NI}))$. Then by proposition (3.8), $NI\text{pint}(A_{NI}) \cup NIsint(A_{NI}) = (A_{NI} \cap NI\text{int}(NI\text{cl}(A_{NI}))) \cup (A_{NI} \cap NI\text{cl}(NI\text{int}(A_{NI}))) = A_{NI} \cap (NI\text{int}(NI\text{cl}(A_{NI})) \cup NI\text{cl}(NI\text{int}(A_{NI}))) = A_{NI}$

Therefore $A_{NI} = NI\text{pint}(A_{NI}) \cup NIsint(A_{NI})$

(b) \Rightarrow (c), By Proposition (3.8) We have,

$$A_{NI} = NI\text{pint}(A_{NI}) \cup NIsint(A_{NI}) = NI\text{pint}(A_{NI}) \cup (A_{NI} \cap NI\text{cl}(NI\text{int}(A_{NI}))) \subseteq NI\text{pint}(A_{NI}) \cup (NI\text{cl}(NI\text{int}(A_{NI}))) = (NI\text{pcl}(NI\text{pint}(A_{NI})))$$

(c) \Rightarrow (a), By Proposition (3.8) We have, $A_{NI} \subseteq NI\text{pint}(A_{NI}) \cup (NI\text{cl}(NI\text{int}(A_{NI}))) \subseteq NI\text{int}(NI\text{cl}(A_{NI})) \cup (NI\text{cl}(NI\text{int}(A_{NI})))$.

Therefore, A is neutrosophic infra b-openset.

Theorem 3.12: Let A_{NI} be anerosophic infra set of a space (X, τ_{NI}) . Then

- (a) $NI\text{bcl}(A_{NI}) = NIscl(A_{NI}) \cap NI\text{pcl}(A_{NI})$
- (b) $NI\text{bint}(A_{NI}) = NIsint(A_{NI}) \cup NI\text{pint}(A_{NI})$

Proof: Obvious.

Theorem 3.13: Let A_{NI} be a neutrosophic infra set of a space (X, τ_{NI}) , then $NIbint (NIbcl (A_{NI})) = NIbcl (NIbint (A_{NI}))$.

Proof: Let A_{NI} be a subset of a space (X, τ_{NI}) .

$$\begin{aligned} \text{Now } NIbint (NIbcl (A_{NI})) &= NIsint (NIbcl (A_{NI})) \cup NIpint (NIbcl (A_{NI})) = \\ NIbcl (NIsint (A_{NI})) \cup NIpint (NIbcl (A_{NI})) \\ &= NIscl (NIsint (A_{NI})) \cup NIpint (NIpcl (A_{NI})) \text{----- (1)} \end{aligned}$$

$$\begin{aligned} \&NIbcl (NIbint (A_{NI})) = NIbcl (NIsint (A_{NI}) \cup NIpint (A_{NI})) = NIbcl (NIsint (A_{NI})) \cup NIbcl (NIpint (A_{NI})) \\ &= NIscl (NIsint (A_{NI})) \cup NIpint (NIpcl (A_{NI})) \text{----- (2)} \end{aligned}$$

Hence, From (1) & (2), $NIbint (NIbcl (A_{NI})) = NIbcl (NIbint (A_{NI}))$

Theorem 3.14:

In a neutrosophic Infra topological space X , every neutrosophic infra b-open set (b-closed set) is a neutrosophic infra β - open set (β -closed set).

Proof: Let A_{NI} be a neutrosophic infra b-open set in X . Then

$$\begin{aligned} A_{NI} \subseteq NIcl (NIint (A_{NI})) \cup NIint (NIcl (A_{NI})) \subseteq NIcl (NIint (NIcl (A_{NI}))) \cup \\ NIint (NIcl (A_{NI})) \subseteq NIcl (NIint (NIcl (A_{NI}))). \end{aligned}$$

Therefore A_{NI} is neutrosophic infra β - open set.

Proposition 3.18: The intersection of a neutrosophic infra-open set and a neutrosophicinfra b – open set is a neutrosophicinfra b – open set.

Proof: The result is obvious.

Theorem 3.19:

Everyneutrosophic infra open set is neutrosophic infra b-open set.

Proof:

Let A_{NI} be a neutrosophic infra open set in (X, τ_{NI}) . Since $A_{NI} \subseteq NIcl(A_{NI})$ and $A_{NI} = NIint(A_{NI})$, $NIint(A_{NI}) \subseteq NIint(NIcl(A_{NI}))$ and then $NIint(A_{NI}) \subseteq NIcl(NIint(A_{NI}))$

which implies that $NI \text{ int}(A_{NI}) \subseteq NI \text{ int}(NIcl(A_{NI})) \cup NIcl(NI \text{ int}(A_{NI}))$. Hence $A_{NI} \subseteq NI \text{ int}(A_{NI}) \subseteq NI \text{ int}(NIcl(A_{NI})) \cup NIcl(NI \text{ int}(A_{NI}))$ and A_{NI} is a neutrosophic infra b-open set in (X, τ_{NI})

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