Neighborhood-prime labeling of Cyclotomic Graph

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Abstract
Let $G = (V(G), E(G))$ be a graph with $n$ vertices. A bijective function $f$ from $V(G)$ to $\{1, 2, \ldots, n\}$ is said to be a neighborhood-prime labeling, if for every vertex $v \in V(G)$ with $\deg(v) > 1$, labels of all the vertices adjacent to $v$ are relatively prime. A graph which admits neighborhood-prime labeling is called a neighborhood-prime graph. In the present work we proved that Cyclotomic Graph $G(n, k)$ is neighborhood-prime graph for some values of $n \& k$.

Keyword: Cyclotomic graph, Neighborhood-prime labeling.

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I. Introduction

S K Patel and N P Shrimali [5] introduced one of the variation of prime labeling which is known as neighborhood-prime labeling of a graph. They proved following graphs are neighborhood-prime: path, complete, wheels, helms, closed helm, flowers, certain union of cycles. S K Patel [7] proved Generalized Petersen graphs are neighborhood-prime graphs for certain cases. Malori Cloys and N Bradley Fox [6] almost covered large Class of trees which have neighborhood-prime labeling such as caterpillars, spiders, firecrackers and any tree that contains no two degree vertices. In addition, Malori Cloys et al. [6] put forth conjecture that all trees are neighborhood-prime, similar conjecture made by Entringer for prime labelings. In [2] J A Gallian published the detailed list of all neighborhood-prime graph.

Definition 1.1: The cyclotomic graph $G(n, k)$ with a positive integer $n > 1$ and integer $k$ such that $0 \leq k \leq n - 1$ is defined to be a graph with $V(G(n, k)) = \{v_i : 1 \leq i \leq n\}$ and $E(G(n, k)) = E_O \cup E_I$ where $E_O = \{v_{2i} : 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor\}$ and $E_I = \{v_1v_{(k+1)}, v_{1+k}v_{(k+1)}v_{1+2(k+1)}, \ldots, v_{n-2k}v_{n-k}, v_{n-k}v_1\}$, here subscripts are taken modulo $n$.

The elements of $E_O$ are called outer edges and the elements of $E_I$ are called inner edges.

Cycle formed by all the inner edges is called inner cycle.

$i_c = \text{The number of inner edges in graph } G(n, k).$

Definition 1.2: For a vertex $v$ in $G$, the neighborhood of $v$ is the set of all vertices in $G$ which are adjacent to $v$ and is denoted by $N(v)$.

Definition 1.3: Let $G = (V(G), E(G))$ be a graph with $n$ vertices. A bijective function $f : V(G) \rightarrow \{1, 2, \ldots, n\}$ is said to be a neighborhood-prime labeling, if for every vertex $v \in V(G)$ with $\deg(v) > 1$, $\gcd\{f(u) : u \in N(v)\} = 1$. A graph which admits neighborhood-prime labeling is called a neighborhood-prime graph.

Definition 1.4: The $n-$sunlet graph is the graph on $2n$ vertices obtained by attaching $n$ pendant edges to a cycle graph $C_n$. 
Here we consider only undirected and non-trivial graphs $G = (V(G), E(G))$ with the vertex set $V(G)$ and edge set $E(G)$. For various graph theoretic notations and terminology we follow Gross and Yellen[3] whereas for number theoretic results we follow D M Burton[1].

II. Main Results

**Theorem 2.1** The $n$-sunlet graph is neighborhood-prime graph.

**Proof:** Let $G$ be $n$-sunlet graph and $v_2,v_4,v_6,...,v_{2n}$ be consecutive vertices of cycle $C_n$ in $G$ and $v_1,v_3,v_5,...,v_{2n-1}$ be pendant vertices of $G$ such that $v_{i+1}$ is an edge where $i$ is odd.

Define $f : V(G) \rightarrow \{1,2,3,...,2n\}$ as $f(v_i) = i$.

We claim that $f$ is a neighborhood-prime labeling.

Here we have $N(v_i) = \{v_{i-1},v_{i+2},v_{i+2}\}$ where $i = 2j, 2 \leq j \leq n$. Now $gcd\{f(v_{i-1}),f(v_{i+2}),f(v_{i+2})\} = gcd\{i-1,i-2,i+2\} = 1$.

Also $N(v_2) = \{v_1,v_4,v_{2n}\}$ and $gcd\{f(v_1),f(v_4),f(v_{2n})\} = 1$ because $f(v_1) = 1$. Therefore, $f$ is a neighborhood-prime labeling.

**Theorem 2.2** The disjoint union of $n$-sunlet graph and finite number of $K_2$ is neighborhood-prime graph.

**Proof:** Let $G$ be a graph consisting of disjoint union of $n$-sunlet graph and finite number of $K_2$.

We know that $n$-sunlet graph is neighborhood-prime graph. Clearly, if $v_i \in V(G)$ and $v_i$ is not vertex of $n$-sunlet graph then $\deg(v_i) = 1$. Therefore, $G$ is neighborhood-prime graph.

**Theorem 2.3** The cycloidal graph $G(n,k)$ is neighborhood-prime graph if $n$ is even and $i_e = \frac{n}{2}$.

**Proof:** From the definition of $G(n,k)$, if $n$ is even and $i_e = \frac{n}{2}$ then $G(n,k)$ is $\frac{n}{2}$-sunlet graph. As $\frac{n}{2}$-sunlet graph is neighborhood-prime graph, $G(n,k)$ is neighborhood-prime graph if $n$ is even and $i_e = \frac{n}{2}$.

**Theorem 2.4** The cycloidal graph $G(n,k)$ is neighborhood-prime graph if $n$ is even and $i_e < \frac{n}{2}$.

**Proof:** From the definition of $G(n,k)$, if $n$ is even and $i_e < \frac{n}{2}$ then $G(n,k)$ is disjoint union of $n$-sunlet graph and finite number of $K_2$. Therefore, $G(n,k)$ is neighborhood-prime graph if $n$ is even and $i_e < \frac{n}{2}$.

**Theorem 2.5** The cycloidal graph $G(n,k)$ is neighborhood-prime graph if $n$ is odd and $i_e < \frac{n}{2}$.

**Proof:** We know that $n$-sunlet graph is neighborhood-prime graph. If we delete one pendant edge from $n$-sunlet graph then the resultant graph $G'$ is also neighborhood-prime graph. Therefore, disjoint union of $G'$ and finite number of $K_2$ is also neighborhood-prime graph. But this disjoint union represents Cycloidal graph $G(n,k)$ when $n$ is odd and $i_e < \frac{n}{2}$. Therefore, $G(n,k)$ is neighborhood-prime graph if $N$ is odd and $i_e < \frac{n}{2}$. 


Theorem 2.6  The cyclotic graph $G(n,1)$ is a neighborhood-prime graph if $n = 6m + 3$ and $n = 6m + 5$ for positive integer $m$.

Proof: Let $G = G(n,1)$ and $v_1, v_2, ..., v_n$ be consecutive $n$ vertices of $G(n,1)$ where $\text{deg}(v_i) = 2$.

Define a bijection $f : V(G) \rightarrow \{1, 2, ..., n\}$ as $f(v_i) = i$. To prove $f$ is a neighborhood-prime labeling we have to show that if $w$ is an arbitrary vertex of $G$ then $gcd\{f(p) : p \in N(w)\} = 1$.

We consider the following three cases.

Case 1: If $w = v_{2i+1}$, $N(v_{2i+1}) = \{v_{2i}, v_{2i-1}, v_{2i+3}\}$ where $i = 1, 2, ..., \frac{n-1}{2}$ and $gcd\{f(v_{2i}), f(v_{2i-1}), f(v_{2i+3})\} = gcd\{2i, 2i-1, 2i+3\} = 1$.

Case 2: If $w = v_{2i}$, $N(v_{2i}) = \{v_{2i+1}, v_{2i+2}, v_{2i-2}\}$ where $i = 1, 2, ..., \frac{n-1}{2}$ and $gcd\{f(v_{2i+1}), f(v_{2i+2}), f(v_{2i-2})\} = gcd\{2i+1, 2i+2, 2i-2\} = 1$.

Case 3: If $w = v_1$ then $N(v_1) = \{v_3, v_{n-1}\}$.

Now $gcd\{f(v_3), f(v_{n-1})\} = gcd(3, n-1) = 1$ because $n = 6m + 3$ or $n = 6m + 5$.

Hence the result follows.

Example:

![Neighborhood-prime labeling of G(9,1)](image1)

Figure 1: Neighborhood-prime labeling of $G(9,1)$

![Neighborhood-prime labeling of G(11,1)](image2)

Figure 2: Neighborhood-prime labeling of $G(11,1)$

Theorem 2.7  The cyclotic graph $G(n,1)$ is a neighborhood-prime graph if $n = 8m + 3$ and $n = 8m + 7$ for positive integer $m$.

Proof: Let $v_1, v_2, ..., v_n$ be consecutive $n$ vertices of $G(n,1)$ where $\text{deg}(v_1) = 2$. Now we rename the vertices of $G(n,1)$ in the following manner.

$v_{4k} = v_k^1$ where $k = 1, 2, ..., \frac{n-3}{4}$

$v_{4k+1} = v_k^2$ where $k = 0, 1, 2, 3, ..., \frac{n-3}{4}$

$v_{4k+2} = v_k^3$ where $k = 0, 1, 2, 3, ..., \frac{n-3}{4}$

$v_{4k+3} = v_k^4$ where $k = 0, 1, 2, 3, ..., \frac{n-3}{4}$

Define a bijection $f : V(G(n,1)) \rightarrow \{1, 2, 3, ..., n\}$ as follows:
In order to show that $f$ is a neighborhood-prime labeling we need to establish that if $w$ is an arbitrary vertex of $G$, then $\gcd\{f(p) : p \in N(w)\} = 1$.

We consider the following nine cases.

**case 1:** If $w = v_0^2$, $N(v_0^2) = \left\{v_0, v_{n-3/4}^3\right\}$

$$\gcd \left\{ f\left(v_0^4\right), f\left(v_{n-3/4}^3\right) \right\} = 1 \text{ because } f\left(v_0^4\right) = 1.$$

**case 2:** If $w = v_0^4$, $N(v_0^4) = \left\{v_0^2, v_0^3, v_0^1\right\}$

$$\gcd \{f(v_0^2), f(v_0^3), f(v_0^1)\} = \gcd\left\{n, n - 1, \frac{n + 1}{2}\right\} = 1.$$

**case 3:** If $w = v_0^3$, $N(v_0^3) = \left\{v_1^4, v_0^4, v_{n-3/4}^3\right\}$

$$\gcd \left\{ f(v_1^4), f(v_0^4), f\left(v_{n-3/4}^3\right) \right\} = 1 \text{ because } f\left(v_0^4\right) = 1.$$

**case 4:** If $w = v_{n-3/4}^4$, $N\left(v_{n-3/4}^4\right) = \left\{v_{n-3/4}^3, v_{n-3/4}^2, v_0^3\right\}$

$$\gcd \left\{ f\left(v_{n-3/4}^3\right), f\left(v_{n-3/4}^2\right), f(v_0^3) \right\} = \gcd\left\{\frac{3n-1}{4}, \frac{3n-5}{4}, n-1\right\} = 1.$$

**case 5:** If $w = v_{n-3/4}^3$, $N\left(v_{n-3/4}^3\right) = \left\{v_{n-3/4}^4, v_{n-3/4}^2, v_0^2\right\}$

$$\gcd \left\{ f\left(v_{n-3/4}^4\right), f\left(v_{n-3/4}^2\right), f(v_0^2) \right\} = \gcd\left\{\frac{n+1}{4}, \frac{n+5}{4}, \frac{n}{4}\right\} = 1.$$

**case 6:** If $w = v_k^1$, $N(v_k^1) = \left\{v_{k-1}^3, v_k^3, v_k^2\right\}$ where $k = 1, 2, \ldots, \frac{n-3}{4}$

$$\gcd \{f(v_{k-1}^3), f(v_k^3), f(v_k^2)\} = \gcd\left\{n-k, n-k-1, \frac{n-1}{2} + k\right\} = 1.$$

**case 7:** If $w = v_k^2$, $N(v_k^2) = \left\{v_{k-1}^4, v_k^4, v_k^1\right\}$ where $k = 1, 2, \ldots, \frac{n-3}{4}$
\[ \gcd \{ f(v_{k-1}^4), f(v_k^4), f(v_k^1) \} = \gcd \left\{ k, k+1, \frac{n+1}{2} - k \right\} = 1. \]

**Case 8:** If \( w = v_k^3 \), \( N(v_k^3) = \{v_{k+1}^1, v_k^1, v_k^4\} \) where \( k = 1, 2, \ldots, \frac{n-7}{4} \)

\[ \gcd \{ f(v_{k+1}^1), f(v_k^1), f(v_k^4) \} = \gcd \left\{ \frac{n-1}{2} - k, \frac{n+1}{2} - k, k+1 \right\} = 1. \]

**Case 9:** If \( w = v_k^4 \), \( N(v_k^4) = \{v_{k+1}^2, v_k^2, v_k^3\} \) where \( k = 1, 2, \ldots, \frac{n-7}{4} \)

\[ \gcd \{ f(v_{k+1}^2), f(v_k^2), f(v_k^3) \} = \gcd \left\{ \frac{n+1}{2} + k, \frac{n-1}{2} + k, n-k-1 \right\} = 1. \]

Hence the result follows.

**Example:**

![Figure 3: Neighborhood-prime labeling of G(19,1)](image)

**Theorem 2.8** \( G(n,1) \) is neighborhood-prime graph if \( n = 8m + 1 \) for positive integer \( m \).

**Proof:** Let \( v_1, v_2, \ldots, v_n \) be consecutive \( n \) vertices of \( G(n,1) \) where \( \deg(v_i) = 2 \). Now we rename the vertices of \( G(n,1) \) following manner.

\[ v_{4k} = v_k^1 \text{ where } k = 1, 2, \ldots, \frac{n-1}{4} \]

\[ v_{4k+1} = v_k^2 \text{ where } k = 0, 1, 2, 3, \ldots, \frac{n-1}{4} \]

\[ v_{4k+2} = v_k^3 \text{ where } k = 0, 1, 2, 3, \ldots, \frac{n-5}{4} \]

\[ v_{4k+3} = v_k^4 \text{ where } k = 0, 1, 2, 3, \ldots, \frac{n-5}{4} \]

Define a bijection \( f : V(G(n,1)) \rightarrow \{1, 2, 3, \ldots, n\} \)
In order to show that \( f \) is a neighborhood-prime labeling we need to establish that if \( w \) is an arbitrary vertex of \( G(n,1) \), then \( \gcd\{f(p) : p \in N(w)\} = 1. \)

We consider the following nine cases.

**case 1:** If \( w = v_0^2 \), \( N(v_0^2) = \left\{ v_0^4, v_{n-1}^2 \right\} \)

\[
gcd\left( f(v_0^4), f\left( v_{n-1}^2 \right) \right) = 1 \text{ because } f(v_0^4) = 1
\]

**case 2:** If \( w = v_0^4 \), \( N(v_0^4) = \{ v_0^2, v_0^3, v_1^2 \} \)

\[
gcd\{f(v_0^2), f(v_0^3), f(v_1^2)\} = gcd\left( n, n-1, \frac{n+1}{2} \right) = 1
\]

**case 3:** If \( w = v_0^3 \), \( N(v_0^3) = \left\{ v_0^4, v_0^4, v_{n-1}^2 \right\} \)

\[
gcd\left( f(v_0^4), f\left( v_0^4 \right), f\left( v_{n-1}^2 \right) \right) = gcd\left( n-1, \frac{n-3}{2}, \frac{3n-3}{4} \right) = 1
\]

**case 4:** If \( w = v_{n-1}^2 \), \( N\left( v_{n-1}^2 \right) = \left\{ v_{n-5}^4, v_{n-1}^4, v_0^3 \right\} \)

\[
gcd\left( f\left( v_{n-5}^4 \right), f\left( v_{n-1}^4 \right), f(v_0^3) \right) = gcd\left( n-1, \frac{n+3}{4}, n-1 \right) = 1
\]

**case 5:** If \( w = v_{n-1}^1 \), \( N\left( v_{n-1}^1 \right) = \left\{ v_{n-5}^3, v_{n-1}^2, v_0^2 \right\} \)

\[
gcd\left( f\left( v_{n-5}^3 \right), f\left( v_{n-1}^2 \right), f(v_0^2) \right) = gcd\left( \frac{3n+1}{4}, \frac{3n-3}{4}, n \right) = 1
\]

**case 6:** If \( w = v_k^2 \), \( N(v_k^2) = \{ v_{k-1}^4, v_k^4, v_k^4 \} \) where \( k = 1, 2, 3, ..., \frac{n-5}{4} \)

\[
gcd\{f(v_{k-1}^4), f(v_k^4), f(v_k^4)\} = gcd\left( k, k+1, \frac{n+1}{2} - k \right) = 1
\]

**case 7:** If \( w = v_k^4 \), \( N(v_k^4) = \{ v_k^3, v_k^2, v_{k+1}^2 \} \) where \( k = 1, 2, ..., \frac{n-5}{4} \)
\[ \text{gcd} \left\{ f(v_{k}^{3}), f(v_{k}^{2}), f(v_{k+1}^{2}) \right\} = \text{gcd} \left\{ n-k-1, \frac{n-1}{2} + k, \frac{n-1}{2} + k + 1 \right\} = 1 \]

**Case 8:** If \( w = v_{k}^{3} \), \( N(v_{k}^{3}) = \{v_{k}^{1}, v_{k}^{2}, v_{k}^{3}\} \) where \( k = 1, 2, \ldots, \frac{n-5}{4} \)

\[ \text{gcd} \left\{ f(v_{k}^{1}), f(v_{k}^{2}), f(v_{k+1}^{1}) \right\} = \text{gcd} \left\{ k+1, \frac{n+1}{2} - k, \frac{n+1}{2} - (k+1) \right\} = 1 \]

**Case 9:** If \( w = v_{k}^{1} \), \( N(v_{k}^{1}) = \{v_{k}^{2}, v_{k-1}^{3}, v_{k}^{3}\} \) where \( k = 1, 2, \ldots, \frac{n-5}{4} \)

\[ \text{gcd} \left\{ f(v_{k}^{2}), f(v_{k-1}^{3}), f(v_{k}^{3}) \right\} = \text{gcd} \left\{ \frac{n-1}{2} + k, n-k, n-k-1 \right\} = 1 \]

Hence the result follows.

**Example:**

![Figure 4: Neighborhood-prime labeling of G(25,1)](image)

**Theorem 2.9** For \( n = 8m + 5 \), \( G(n,1) \) is neighborhood-prime graph if \( n = 1 \pmod{6} \).

**Proof:** Let \( v_{1}, v_{2}, \ldots, v_{n} \) be consecutive \( n \) vertices of \( G(n,1) \) where \( \text{deg}(v_{1}) = 2 \). Now we rename the vertices of \( G(n,1) \) following manner.

\[
\begin{align*}
v_{4k} &= v_{k}^{1} \quad \text{where} \quad k = 1, 2, \ldots, \frac{n-1}{4} \\
v_{4k+1} &= v_{k}^{2} \quad \text{where} \quad k = 0, 1, 2, 3, \ldots, \frac{n-1}{4} \\
v_{4k+2} &= v_{k}^{3} \quad \text{where} \quad k = 0, 1, 2, 3, \ldots, \frac{n-5}{4} \\
v_{4k+3} &= v_{k}^{4} \quad \text{where} \quad k = 0, 1, 2, 3, \ldots, \frac{n-5}{4}
\end{align*}
\]

Define a bijection \( f : V(G(n,1)) \rightarrow \{1, 2, 3, \ldots, n\} \)
In order to show that $f$ is a neighborhood-prime labeling we need to establish that if $w$ is an arbitrary vertex of $G(n, 1)$, then $gcd\{f(p) : p \in N(w)\} = 1$.
We consider the following eight cases:

**case 1:** If $w = v_0^2$, $N(v_0^2) = \left\{ \frac{4}{y_0}, \frac{1}{y_0}, \frac{4}{y_n-1} \right\}$

Now $gcd\left\{ f(v_0^2), f(v_n^{-1}) \right\} = gcd\left( \frac{n+3}{4}, \frac{3n+5}{4} \right)$

we have to show that $d = gcd\left( \frac{n+3}{2}, \frac{3n+5}{4} \right) = 1$

Now $d = gcd\left( \frac{n+3}{2}, \frac{3n+5}{4} \right) = gcd(4m+4, 6m+5)$, since $n = 8m+5$.

Therefore, $d \mid (3(4m+4) - 2(6m+5))$, so we get $d \mid 2$. Hence $d = 1, 2$.
But 2 does not divide $6m+5$. Hence $d = 1$.

**case 2:** If $w = v_0^3$, $N(v_0^3) = \left\{ \frac{1}{y_1}, \frac{4}{y_0}, \frac{2}{y_n-1} \right\}$

$gcd\left\{ f(v_1), f(v_0^4), f\left( \frac{2}{y_n-1} \right) \right\} = gcd\left( \frac{n+3}{2}, \frac{n+3}{4} \right)$

We have $n = 8m+5$ such that $n = 1 \mod 6$. Therefore, there exist positive integer $k$ such that $n = 6k + 1$. Therefore, $n = 6k + 1 = 8m + 5$ for some $k$, gives $3k - 4m = 2$ and hence we get $m = 3t + 1$ and $k = 4t + 2$ for non negative integer $t$.
So finally we get $n = 24t + 13$.

Now we have to show that $d = gcd\left( \frac{n+3}{2}, n \right) = gcd(12t+8, 24t+13) = 1$.

Now $d \mid (2(12t+8) - (24t+13))$, so we get $d \mid 3$. Hence $d = 1, 3$.
But 3 does not divide $12t + 8$. Hence $d = 1$.

**case 3:** If $w = v_{n-1}^2$, $N(v_{n-1}^2) = \left\{ \frac{4}{y_{n-5}}, \frac{1}{y_{n-1}}, \frac{3}{y_0} \right\}$

$gcd\left\{ f\left( \frac{4}{y_{n-5}} \right), f\left( \frac{1}{y_{n-1}} \right), f\left( \frac{3}{y_0} \right) \right\} = gcd\left( \frac{3n+1}{4}, \frac{3n+5}{4}, \frac{n+1}{2} \right) = 1$

**case 4:** If $w = v_{n-1}^3$, $N(v_{n-1}^3) = \left\{ \frac{3}{y_{n-5}}, \frac{2}{y_{n-1}}, \frac{2}{y_0} \right\}$
gcd \left\{ f\left(\frac{3}{n-5}\right), f\left(\frac{2}{n-1}\right), f\left(v_0^2\right) \right\} = gcd \left\{ \frac{n+7}{4}, \frac{n+3}{4}, 1 \right\} = 1

case 5: If \( w = v_k^1 \), \( N(v_k^1) = \{v_k^2, v_{k-1}^3, v_k^3\} \) where \( k = 1, 2, ..., \frac{n-5}{4} \)

gcd\{f(v_k^2), f(v_{k-1}^3), f(v_k^3)\} = gcd \left\{ k + 1, \frac{n+1}{2} - (k-1), \frac{n+1}{2} - k \right\} = 1

case 6: If \( w = v_k^2 \), \( N(v_k^2) = \{v_{k-1}^4, v_k^4, v_k^1\} \) where \( k = 1, 2, 3, ..., \frac{n-5}{4} \)

gcd\{f(v_{k-1}^4), f(v_k^4), f(v_k^1)\} = gcd \left\{ \frac{n+3}{2} + k - 1, \frac{n+3}{2} + k, n - k + 1 \right\} = 1

case 7: If \( w = v_k^3 \), \( N(v_k^3) = \{v_k^4, v_k^1, v_{k+1}^1\} \) where \( k = 1, 2, ..., \frac{n-5}{4} \)

gcd\{f(v_k^4), f(v_k^1), f(v_{k+1}^1)\} = gcd \left\{ \frac{n+3}{2} + k, n - k + 1, n - k \right\} = 1

case 8: If \( w = v_k^4 \), \( N(v_k^4) = \{v_k^3, v_k^2, v_{k+1}^2\} \) where \( k = 0, 1, 2, ..., \frac{n-5}{4} \)

gcd\{f(v_k^3), f(v_k^2), f(v_{k+1}^2)\} = gcd \left\{ \frac{n+1}{2} - k, k + 1, k + 2 \right\} = 1

Hence the result follows.

Example:

![Figure 5: Neighborhood-prime labeling of G(13,1)](image)

Conclusion and Future Scope:
It may be interesting to find neighborhood-prime labeling of union of cyclotic graphs. Also one can think about the neighborhood-prime labeling of one point union of cyclotic graphs.

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